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Control of Lightly Damped, Flexible Modes in the Controller Crossover Region

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Many proposed future space structures have closely spaced, lightly damped flexible modes that must actively be controlled to achieve desired performance objectives. These densely packed structural modes require a controller to roll off in a region of modes that have some amount of modeling error or uncertainty in their damping levels, natural frequencies, and/or mode shapes. Therefore, a delicate balance must be found between the level of closed-loop performance of the control system and its robustness to various types of uncertainty. This paper focuses on using the structured singular value (μ) control design technique to achieve performance and robustness objectives in the presences of uncertainty flexible modes in the controller crossover region. Controllers are synthesized for a lightly damped, flexible structure experiment with noncollocated sensors and actuators to show the role played by uncertainty modeling in achieving desired performance objectives.

Nomenclature

D_Δ = μ lower-bound scaling matrix
Q_Δ = μ upper-bound scaling matrix
Δ = perturbation block structure
μ = structured singular value
ρ = spectral radius
$\bar{\sigma}$ = maximum singular value
ω = frequency variable
$\|\cdot\|_\infty$ = infinity norm

Introduction

STRINGENT requirements envisioned on the pointing and shape accuracy of future space missions necessitate advances in the control of flexible structures. These structures will be extremely flexible and have little natural damping and their modes will be densely packed throughout the frequency domain of interest. The goals of the mission often require a controller to attenuate vibration in one frequency range but not destabilize structural modes outside this range.[1] This control problem is made more difficult by inaccuracies and modeling errors in the structural models.

The synthesis of controllers for lightly damped, flexible structures that achieve desired performance objectives as well as being robust to modeling errors has been of interest for a number of years. A difficulty has been to address the controller synthesis issue in a straightforward manner for structures with either collocated or noncollocated sensors and actuators. A number of researchers have observed that neglecting unmodeled dynamics or the control and observation spillover phenomena from unmodeled modes may lead to instability in the closed-loop system.[2–4] Several proposed methods for eliminating spillover effects rely on direct velocity output feedback, a large gap between the resonant modes being controlled, and the unmodeled modes or the placement of

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sensors and actuators to reduce the effect of unmodeled structural modes.[2,3,5] This problem was also addressed by designing low and high authority controllers to suppress certain modes and reduce the amount of spillover.[6] The high authority controller was synthesized by frequency shaping of the linear quadratic Gaussian (LQG) cost functional.

This paper proposes a straightforward method to controller synthesis, based on the structured singular value (μ), which achieves desired performance objectives and is robust to both uncertain modes in the crossover region and higher frequency modes. The approach makes use of frequency weighting functions in the μ framework and is independent of the collocation of the actuators and sensors. To validate this method in the laboratory, controllers are synthesized for a lightly damped, experimental structure with noncollocated sensors and actuators.[7–9] The closed-loop specifications on the experimental structure require the controller to crossover between two closely spaced, lightly damped modes. One set of lightly damped modes is actively controlled.

Flexible Structure Experiment

An experimental flexible structure was designed at California Institute of Technology to exhibit a number of attributes associated with large flexible space structures.[8,9] These included closely spaced, lightly damped modes, noncollocated sensors and actuators, and numerous modes in the controller crossover region. In addition to these considerations, expendability of the structure was a desired feature. Expandability provides a means for increasing the modal density in a frequency range of interest.

The initial experimental structure, Fig. 1, consists of two stories, three longerons (columns), and three noncollocated sensors and actuators. The first story columns are 0.838 m long with the second story columns measuring 0.759 m. Including the platforms, the structure has a height of 1.651 m. The two platforms are triangular in shape with a 0.406-m base and a height of 0.353 m. A longeron is connected to a platform by a triangular mating fixture and three bolts. The entire structure hangs from a mounting structure fixed to the ceiling. The three actuators are attached to the mounting structure and act along the diagonals of the first story.

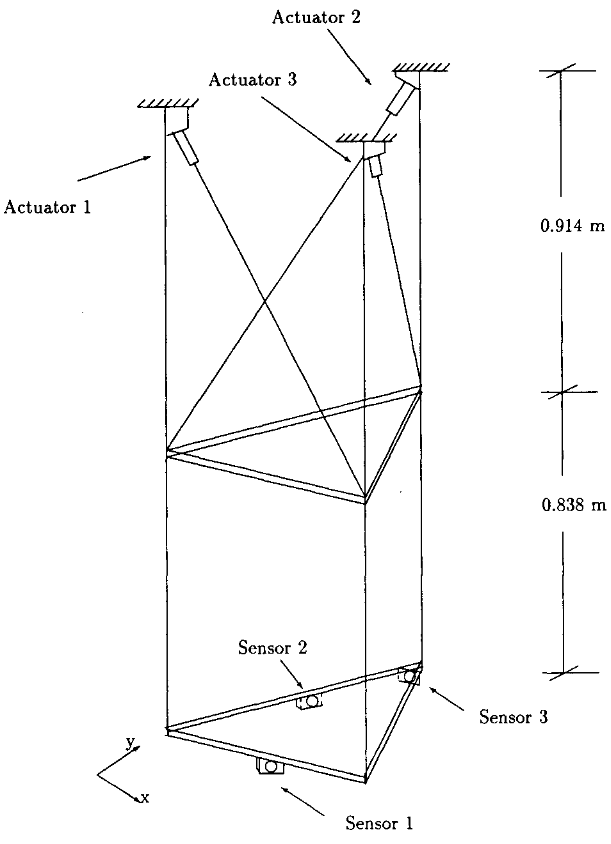

Fig. 1 Caltech experimental flexible structure.

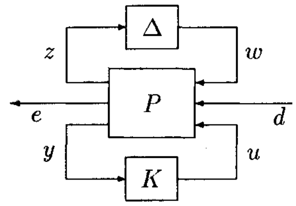

Fig. 2 General interconnection structure.

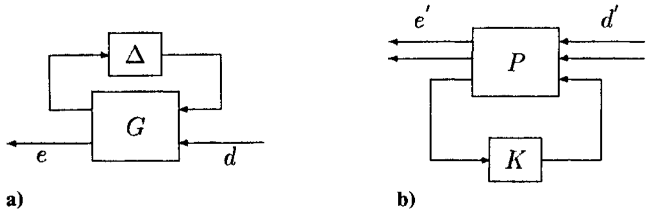

Fig. 3 Control a) analysis and b) synthesis problem.

The actuators are a voice coil type design, built by Northern Magnetics Inc., which output a force proportional to an input voltage. The actuators are rated at ± 13.3 N of force at ± 5 V and have a bandwidth of 60 Hz. The sensors are Sunstrand QA-1400 accelerometers. These are mounted on the second story platform, located along the x axis, y axis and at 45 deg to both axes. The accelerometers are extremely sensitive and have a flat frequency response between 0 and 200 Hz. The noise associated with them is rated at 0.05% of the output at 0–10 Hz and 2% at 10–100 Hz. The sensors are scaled for accelerations of 0.016 g per volt, providing a maximum ± 5-V output at peak accelerations of the input disturbance. The accelerometer output is conditioned by a 40-Hz, fourth-order Butterworth filter prior to input into the analog-to-digital (A/D) converter to attenuate the response of high frequency modes of the structure and measurement noise.

The closed-loop design is implemented via a 5400 Masscomp computer. The real time controller has sample rate of 200 Hz. The computer speed is such that a three input, three output, 90th-order control law, in modal coordinates, can be implemented. The 12 bit A/D converter has a range of $\pm$ 5 V, 0.00244 V per bit, along with a 12-bit digital-to-analog (D/A) converter with a range of $\pm$ 5 V. The noise associated with the computer is $\pm$ 1 least significant bit (lsb). The Masscomp computer is entirely dedicated to the closed-loop experiment during real time implementation.

Structured Singular Value (μ)

This section briefly reviews the frequency-domain methods for analyzing the performance and robustness properties of feedback systems using structured singular value (μ).[10–13] The general framework used in the paper is shown in Fig. 2. Any linear interconnection of inputs, outputs, and commands along with perturbations and a controller can be viewed in this context and rearranged to match Fig. 2.

Definitions

Nominal stability (NS): the nominal plant model has to be stabilized by the controller design.

Nominal performance (NP): performance is defined in terms of the weighted H_∞ norm of the closed-loop system between the exogenous inputs (disturbances) and "errors" (sensor outputs). This norm is defined as the worst case closed-loop response, over frequency, to disturbances.

Robust stability (RS): the closed-loop system must remain stable for all plant models as defined by the uncertainty descriptions.

Robust performance (RP): the closed-loop system must satisfy the performance requirement for all plant models as defined by the uncertainty description.

Most modern control design methods only address the problem of nominal stability and nominal performance. The stability margins used in classical frequency domain methods attempt to address the robust stability problem, but these margins may be misleading for multivariable systems. One method that deals with the robust performance question is μ-based analysis and synthesis techniques.

Analysis Overview

For the purpose of analysis, the controller may be thought of as just another system component. The inclusion of the controller into the plant reduces the diagram in Fig. 2 to that in Fig. 3a. The analysis problem involves determining whether the output error signal e remains small for a given set of inputs d and norm-bounded perturbations Δ. These perturbations Δ describe errors or uncertainties in the mathematical model. This requires that all weighting functions and scalings used to describe such perturbations be absorbed into the open-loop interconnection structure P. The closed-loop interconnection structure G, which includes that controller K, is partitioned such that the input-output map from d to e is the following linear fractional transformation (LFT):

$$e = F_u(G, \Delta)d$$

where

$$F_u(G, \Delta) = G_{22} + G_{21}\Delta(I - G_{11}\Delta)^{-1}G_{12}, \quad G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

Nominal performance in this formulation is simply a norm test on the transfer matrix G_{22}, with ω representing the frequency variable.

$$\|G_{22}\|_\infty = \sup_\omega \bar{\sigma}\,[G_{22}(jw)]$$

One can see this directly by setting the model uncertainty Δ in the problem formulation to zero. Robustness to unstructured uncertainty corresponding to the perturbations Δ as a single full block [only $\bar{\sigma}(\Delta) \le 1$ known] depends only on an infinity norm test on G_{11}. Unfortunately, norm bounds are inadequate for dealing with the question of robust performance or realistic descriptions of plant uncertainty. The structured singular value μ, which is a generalization of a singular value, is used to determine robustness of the closed-loop system to structured modeling uncertainty and the achievable performance level in the presence of uncertainty.

For simplicity only complex perturbations will be considered. First, assume that Δ belongs to the set

$$\boldsymbol{\Delta} = \{\mathrm{diag}(\delta_1 I_{k_1}, \ldots, \delta_r I_{k_r}, \Delta_1, \Delta_2, \ldots, \Delta_s) : \delta_i \in \mathcal{C}, \Delta_i \in \mathcal{C}^{n_i \times m_i}\}$$

$$B\Delta = \{\Delta \in \boldsymbol{\Delta} \mid \bar{\sigma}(\Delta) \le 1\}$$

The function μ is defined as

$$\mu_\Delta(M) = (\min_{\Delta \in \boldsymbol{\Delta}}\{\bar{\sigma}(\Delta) : \det(I - \Delta M) = 0\})^{-1}$$

$$\mu_\Delta(M) = 0 \quad \text{if no } \Delta \in \boldsymbol{\Delta} \text{ solves } \det(I - \Delta M) = 0$$

and has the property that $\mu_\Delta(\alpha M) = |\alpha| \mu_\Delta(M)$. The μ is a function of M, a constant complex matrix, which depends on the structure of Δ. The structured singular value μ is difficult to compute, hence upper and lower bounds have been derived that involve two sets of block diagonal scaling matrices. These scaling matrices are dependent on the underlying block structure, which is directly associated with perturbation block structure Δ. Let

$$\boldsymbol{Q}_\Delta = \{Q \in \boldsymbol{\Delta} : Q^*Q = I_n\}$$

$$\boldsymbol{D}_\Delta = \{\mathrm{diag}(D_1, \ldots, D_r, d_1 I_{n_1 \times n_1}, \ldots, d_1 I_{n_r \times n_r}) : D_i = D_i^* \in \mathcal{C}^{r_i \times r_i}, D_i > 0, d_i \in \mathfrak{R}, d_i > 0\}$$

where the sets $\boldsymbol{Q}_\Delta$ and $\boldsymbol{D}_\Delta$ match the structure of Δ. Note that $\boldsymbol{Q}_\Delta$ and $\boldsymbol{D}_\Delta$ leave Δ invariant in the sense that $\Delta \in \boldsymbol{\Delta}$, and $D \in \boldsymbol{D}$ implies that $\bar{\sigma}(\Delta Q) = \bar{\sigma}(Q\Delta)$ and $D\Delta D^{-1} = \Delta$. The sets $\boldsymbol{Q}_\Delta$ and $\boldsymbol{D}_\Delta$ are used to obtain the following upper and lower bounds for μ (Refs. 10–13).

$$\sup_{Q \in \boldsymbol{Q}_\Delta} \rho(MQ) \le \mu(M) \le \inf_{D \in \boldsymbol{D}_\Delta} \bar{\sigma}(DMD^{-1})$$

These bounds form the basis of computation techniques for the μ problem.[13–15]

Key theorems regarding μ prove that the lower bound is always an equality and the upper bound is an equality for three or fewer complex full blocks. Unfortunately, the optimization problem implied by the lower bound has multiple local maxima and therefore does not immediately yield a reliable computational approach. The upper bound $\bar{\sigma}(DMD^{-1})$ is convex, though the infimum is not necessarily equal to μ.[11–13] The upper and lower bounds for μ are not guaranteed to be "tight," in general, but in fact are usually tight for problems of engineering interest. Practical implementations of these bounds have been developed, and efficient software is available for their computation.[16]

The importance of μ in studying robustness of feedback systems is due to the following two theorems, which characterizes robust stability and robust performance of a system with structured uncertainty.

Theorem 1: Robust Stability. The linear fractional transformation of the closed-loop system and the perturbation structure $F_u(G\ \Delta)$, is stable for all $\Delta \in B\Delta$ if and only if the suprememum over all frequency ω of the structure singular value of G_{11}, $\mu\,[G_{11}(jw)]$, is less than 1.

Theorem 2: Robust Performance. The linear fractional transformation of the closed-loop system and the perturbation structure $F_u(G, \Delta)$ is stable for all $\Delta \in B\Delta$ and its infinity norm is less than 1, $\|F_u(G, \Delta)\|_\infty \le 1$, if and only if the suprememum over all frequency ω of the structure singular value of G, $\sup_\omega \mu[G(jw)]$ is less than 1. The μ is computed with respect to the structure $\boldsymbol{\Delta} = \{\mathrm{diag}(\Delta, \Delta_{n+1}), \Delta \in \boldsymbol{\Delta}\}$.

Theorem 1 implies that stability of any linear, time-invariant system in the presence of structured uncertainty can be rewritten as a structured singular value test. Theorem 2 states, given a performance objective described in terms of an infinity norm test, robust performance of any linear, time-invariant system in the presence of structured uncertainty can be written as a structured singular value test.

H_∞ Controller Synthesis Review

The perturbation block Δ can be normalized to 1 and the normalizing factor absorbed into P. This results in the synthesis problem as shown in Fig. 3b. Hence, the controller synthesis problem involves finding a stabilizing controller K such that the performance requirements are satisfied under prescribed uncertainties. The interconnection structure P is partitioned so that the input-output map from d' to e' is expressed as the linear fractional transformation $e' = F_l(P,K)d'$ where $F_l(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1} P_{21}$. For the H_∞ optimal control problem, the objective is to find a stabilizing controller K that minimizes $\|F_l(P, K)\|_\infty$. A detailed review of the H_∞ control problem is given in Ref. 17, and state-space results are discussed in Refs. 18 and 19.

μ-Synthesis Control Design: *D-K* Iteration

The μ-synthesis methodology is a practical approach to designing control systems to achieve robust performance objectives. This technique integrates two powerful theories for synthesis and analysis into a systematic control design technique involving H_∞ optimization methods for synthesis and the structured singular value for analysis. Recall that an upper bound for μ may be obtained by scaling and applying the infinity norm. Extending this concept to synthesis, the problem of robust controller design becomes that of finding a stabilizing controller K and scaling matrix D such that the quantity $\|DF_l(P, K)D^{-1}\|_\infty$ is minimized.

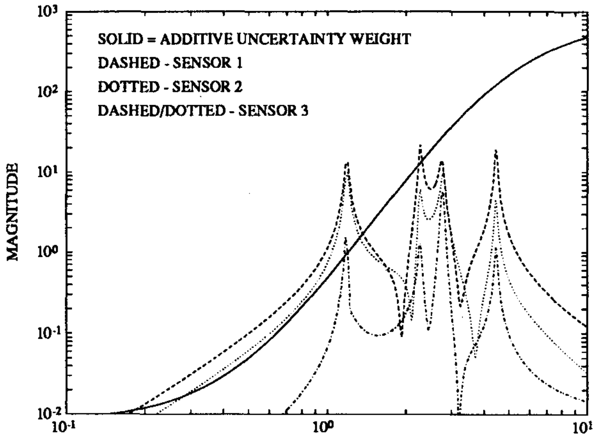

Fig. 4 Experimental transfer functions from voice coil actuator 1 to sensors 1–3 and additive weight.

Table 1 Damping ratios and natural frequencies of the experiment

	Experimental		
Mode	Natural frequency, Hz	Damping ratio, %	Mode type
1	1.17	1.8	First X bending
2	1.19	1.8	First Y bending
3	2.26	1.0	First torsional
4	2.66	1.6	Second X bending
5	2.75	1.8	Second Y bending
6	4.43	0.9	Second torsional

One approach to minimizing the quantity $\|DF_l(P,K)D^{-1}\|_\infty$ is to alternately minimize the expression for K or D while holding the other constant. For fixed D, $\|DF_l(P,K)D^{-1}\|_\infty$ becomes an H_∞ optimal control problem and can be solved using the well-known state-space method. With fixed K, it is just a μ-analysis problem whose upper bound can be minimized at each frequency using convex optimization methods. The resulting D scaling data, which is a function of frequency, is fit with an invertible, stable, minimum-phase, real-rational transfer function and wrapped back into the open-loop interconnection structure P. This process is carried out iteratively until a satisfactory controller is constructed. Although this iterative scheme is not guaranteed to find a global optimum of the preceding minimization problem, controllers exhibiting good performance and robustness have been obtained for a variety of applications. A more detailed discussion of D-K iteration can in found in Refs. 16 and 20.

The μ-synthesis techniques have been used extensively to synthesize robust controllers for flexible structures. Controllers have been formulated for the vibration attenuation problem and implemented on a number of flexible structures experiments.[9,21,22] The control designs synthesized using μ synthesis achieved a high level of vibration attenuation along with good robustness characteristics. The μ-synthesis methodology is employed to address the design of controllers that achieve robustness and performance objectives while crossing over the control design in the presence of numerous flexible modes.

Structural Modeling

A model of the structure relating input signals to outputs is derived experimentally for control design. A filtered, white noise random process is input to each voice-coil actuator and the accelerations due to this signal are measured by the sensors. A sample rate of 200 Hz is used for the identification experiments, the same as in the closed-loop control experiments. Each single-input to multi-output identification experiment is run for a total of 409.6 s (81,920 sample points). A Fourier transform of the time history is performed on each input/output pair. A total of nine transfer functions are determined. Figure 4 shows the transfer function associated with voice-coil actuator 1 and the three sensor outputs. The first group of local modes involve bending of the longerons and diagonals and occur between 37 and 42 Hz.

The multivariable model of the experimental structure developed from the experimental data accurately represents the input/output behavior of the real system.[9,23] It contains six modes and has four inputs and three outputs. The six modes have the same natural frequencies and damping values as the experimental structure (see Table 1). This model is used to synthesize controllers for this paper.

Control Objectives

The control objective is to attenuate vibration of select modes of the structure at the accelerometer locations. The structure has an input disturbance due to an air actuator blowing directly on sensor one. The disturbance takes the form of a sinusoid sweep through the frequency range of 1–6 Hz. The performance requirement is to minimize the maximum frequency response of the first two bending modes of the closed-loop system in comparison with their open-loop response. This is to be achieved contiguous with being robust to unmodeled higher frequency modes present in the structure.

Control Problem Formulation

The μ-synthesis control design methodology requires the nominal model of the structure to be described as a linear time invariant (LTI) system. Although this model may describe the physical system accurately, any model is only an approximation to the physical system. There is always some modeling error or "uncertainty" present even when the underlying process is essentially linear. These uncertain physical parameters may be due to neglected high frequency dynamics or invalid assumptions made in the model formulation. Inaccuracies in modeling may be described in numerous ways, such as bounds on the parameters of a linear model, bounds on the nonlinearities, and frequency domain bounds on transfer function models.

Uncertainty descriptions determine the tradeoff between achievable performance and robustness of the control design. A controller synthesized for a model with no assumed modeling error will theoretically achieve a higher level of performance than one designed to account for modeling error. A controller synthesized for a physical system that is not in the set of plants described by the nominal and uncertainty models may be unstable or exhibit poor performance when implemented on the actual system. However, if the uncertainty descriptions are overly conservative, models may be included in the set that severely limit the performance of the closed-loop system. Therefore, tight uncertainty bounds are required to provide robust control designs that achieve high performance when implemented on the actual system.

A frequency domain description of uncertainty is used in this study to describe modeling errors and robustness objectives in the control design model. The flexible structure experiment is described by a nominal LTI system and frequency varying uncertainty models in the control problem formulation. The uncertainty models are developed to account for the variation between the structural model and experimental data. The level of uncertainty may vary as a function of frequency. For example, several input-output experiments are performed and experimental Bode plots of the system are derived. These plots are compared to the theoretical model of the structure and bounds on the variation in magnitude, and phase between the experimental data and the mathematical model are derived as a function of frequency. Variations between the experimental data and model are approximated by disk-shaped regions. These regions are described by multiplicative and additive uncertainty weights in the problem formulation.[9,12]

The nominal plant model together with the frequency domain uncertainty models define a set of plants within which the real physical system is assumed to lie. The μ-synthesis methodology

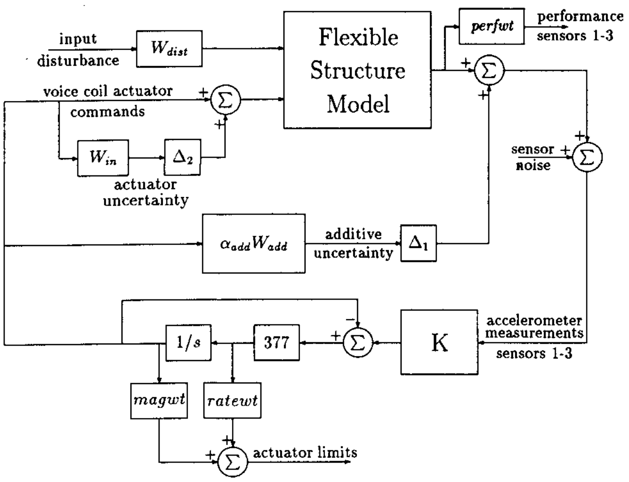

Fig. 5 Block diagram of control problem.

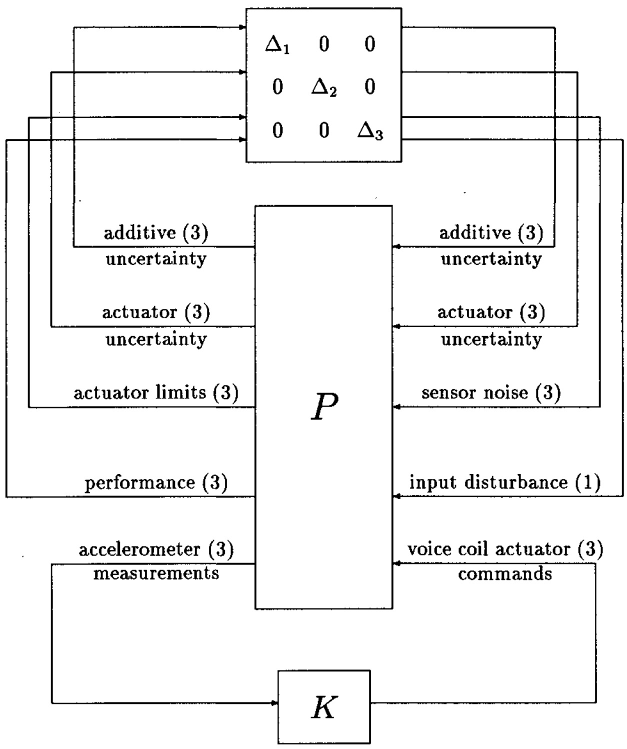

Fig. 6 Interconnection structure for μ-synthesis control design.

uses this information to optimize the control design and achieve robustness and performance objectives.[16,20] An additive uncertainty W_{add} is included in the control problem formulation to account for unmodeled high frequency dynamics of the structure and most importantly, to smoothly transition between controlled and, uncontrolled structural modes.This weight is selected to have a low magnitude at low frequency to reflect that the nominal model accurately represents the real system at low frequency. The additive uncertainty weight is rolled up rapidly to ensure that the crossover region of the controller does not destabilize any modes. The weight takes on a peak magnitude that envelopes the modes to be gain stabilized. Selection of the additive uncertainty weight is critical in the μ-synthesis control problem formulation to ensure both the performance and robustness objectives are achieved.

Experiment Uncertainty Descriptions

The first two bending modes of the experimental structure are actively attenuated to achieved desired performance specifications. This requires the controller to roll off between the first two bending modes at 1.19 Hz and the first torsional mode at 2.26 Hz. The additive uncertainty weight is selected to encompass the frequency domain peak of the first torsional mode and higher frequency modes. The modes enveloped by the additive uncertainty weight are to be gain stabilized by the synthesized controller. A plot of the additive uncertainty weight w_{add} and transfer functions from voice coil actuator 1 to the three accelerometer outputs is shown in Fig. 4.

The additive uncertainty weight bounds the maximum gain of the torsional mode and treats the phase component as being totally unknown. This uncertainty description will require the *D-K* iteration procedure to roll off the controller between 1.2 and 2.2 Hz in the experiment to gain stabilize the first torsional and higher modes. Several control designs are synthesized with the magnitude of the additive uncertainty weight allowed to vary to gauge the effect of the additive weight magnitude on the closed-loop performance and robustness. The additive weight used is

$$w_{add} = 800\,\frac{(s+3)^5}{(s+30)^5} \qquad W_{add} = w_{add}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0.25\end{bmatrix} \tag{1}$$

The weight associated with sensor 3 is scaled by 0.25 due to the lower gain in the corresponding experimental transfer function data. A constant input multiplicative uncertainty weight *W* of 4% is used in the problem formulation to account for actuator and mode shape errors. The value of this weight is based on results cited in Ref. 21. The input multiplicative uncertainty weight is selected to be constant since it only plays a role in the control design at the first two bending modes. Above and below the first bending modes, the additive uncertainty weight dominates the control problem. A constant multiplicative uncertainty weight aids in reducing the number of states in the control problem.

Problem Setup

Controllers are designed using the μ-synthesis methodology to address the issue of crossing over the closed-loop control between numerous flexible modes. The control design must be robust to unmodeled structural modes as well as attenuate the response of the first two bending modes close to the unmodeled modes. To achieve this goal, an additive uncertainty weight W_{add} is included in the control problem formulation. An input uncertainty *W* is included in the problem to account for inaccuracies between the transfer function model and the experimental data.

The performance requirements are to attenuate vibration of the first two bending modes although not exceed limits on the actuator force and rate. Frequency domain weights are included on the actuator magnitude and rate commands to limit their response. These weights perform a similar function to the quadratic control effort weight *R* in the standard LQG control problem. They are used to effectively penalize controller requests for large magnitude forces and fast response speed. The magnitude and rate weights must be scaled correctly to correspond to the magnitude of the input disturbance effecting the system. The actuator magnitude weight is determined by its maximum output of ± 13.3 N and the input force disturbance level (magwt = 1/80). The actuator rate weight corresponds to the 60-Hz bandwidth of the voice coil actuators and the input force disturbance level (ratewt = 1/3770). The accelerometer measurements are modeled to include sensor noise. The sensor noise level is based on data derived from the manufacturer's specifications and is modeled as a constant 2.5×10^{-3}. A block diagram of the control problem formulation is shown in Fig. 5.

The vibration attenuation objective is to minimize the maximum response of the first two bending modes in the frequency domain. This is similar to adding damping to the first two bending modes. To achieve this, the performance weight for vibration attenuation is selected as a constant scaling on each sensor transfer function outputs. The peak of the first two bending modes is first scaled to 1, then a constant performance weight (perfwt) is used to deter-

Table 2 Parameters used in control design and experimental results

Design	Structural model, modes	Designed performance	Additive scaling, α_{add}	Experiment performance
$K1$	6	0.36	1.0	0.20
$K2$	6	0.21	0.5	0.13
$K3$	6	0.14	0.1	0.09
$K4$	2	0.37	1.0	0.22
$K5$	2	0.21	0.5	0.13
$K6$	2	0.16	0.1	0.11

mine the amount of attenuation of the frequency domain peaks. Scaling the peaks of the three disturbance to accelerometer transfer functions to 1 requires the individual transfer functions to be scaled by 0.33, 0.50, and 2.78, respectively. For example, a performance weight of 5, perfwt = 5*diag(0.33, 0.50, 2.78), corresponds to the closed-loop system having a fifth of the peak frequency response of the open-loop system.

The input disturbance used to excite the structure enters via an air actuator blowing directly on sensor 1. A model of the disturbance to excitation transfer function was experimentally derived. The input disturbance and air actuator are modeled as a first-order 10-rad/s filter, $W_{dist} = 10/(s + 10)$.

The block diagram in Fig. 5 is redrawn as an LFT to fit into the μ-synthesis framework as shown in Fig. 6. The dimensions of the complex uncertainty blocks are 3×3 for Δ_1, 3×3 for Δ_2, and 6×4 for Δ_3. Δ_1 is associated with the additive uncertainty, Δ_2 with the actuator uncertainty, and Δ_3 is the performance block. All of the Δ blocks are full blocks. The D-K iteration procedure is used to synthesize controllers for the experimental flexible structure that roll off between closely spaced damped modes.

Robust stability of the closed-loop system is achieved if μ of the corresponding system is less than 1 with respect to perturbations Δ_1 and Δ_2. Robust performance is achieved if μ of the closed-loop system is less than 1 with respect to perturbations Δ_1, Δ_2, and Δ_3.

Control Designs

Two sets of three controllers are formulated for the flexible structure experiment. The interconnection structure for the first set of three controllers synthesized, $K1$, $K2$, and $K3$, uses the complete, experimentally derived, six-mode structural model of the experiment. The second set of three controllers, $K4$, $K5$, and $K6$, are synthesized using only the first two bending modes in the nominal structural model. For each of the three controllers designed for the two different nominal structural models, there is a one-to-one correspondence between the uncertainty descriptions (W_{in},W_{add}) and performance weights (W_{dist}, magwt, ratewt, and sensor noise) except for a difference in the performance weight associated with the attenuation of the first two bending modes, perfwt.

The three controllers are formulated for varying levels of additive uncertainty for each nominal model to determine its affect on the closed-loop performance and robustness. Table 2 lists the parameters used in each design. In the synthesis of each control design, the vibration attenuation level (perfwt) is scaled to achieve a μ value of 1 for the closed-loop system. Robust performance μ plots for controllers $K1$, $K2$, $K3$, $K4$, $K5$, and $K6$ are shown in Fig. 7. The robust stability and performance μ plots of controllers $K5$ and $K6$ implement on the six-mode, structural model are shown in Fig. 8. Figure 9 contains loop gain plots of controllers $K4$ and $K6$.

Experimental Results

Performance is measured as a ratio of closed-loop frequency domain response to the open-loop response for the first two bending modes in the frequency range of 0.5 and 2.0 Hz. Results of the experiments are shown in Table 2. Controllers $K1$ through $K5$ attenuate the frequency domain peaks of the first two bending modes but do not destabilize the other structural modes. In the case of the three controllers designed with the full six-mode model of the

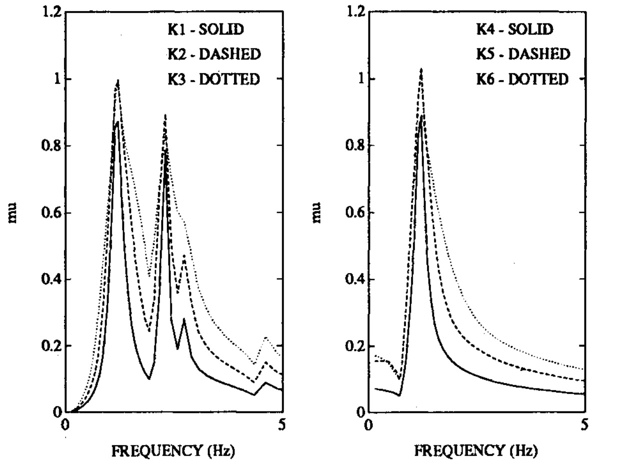

Fig. 7 Plots of μ for controllers $K1$, $K2$, $K3$, $K4$, $K5$, and $K6$.

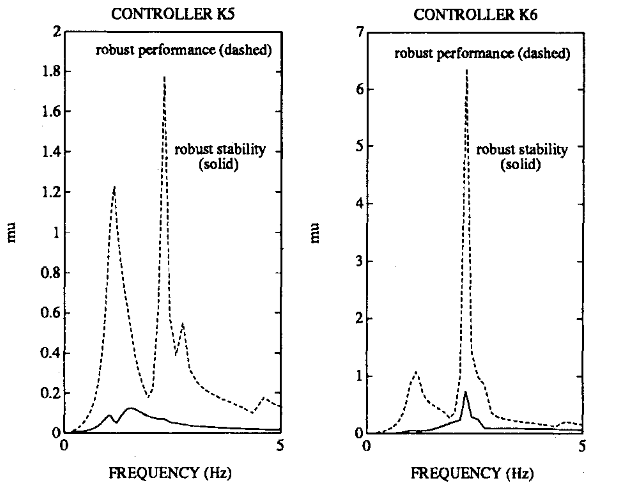

Fig. 8 Robust stability and performance μ plots of controllers $K5$ and $K6$.

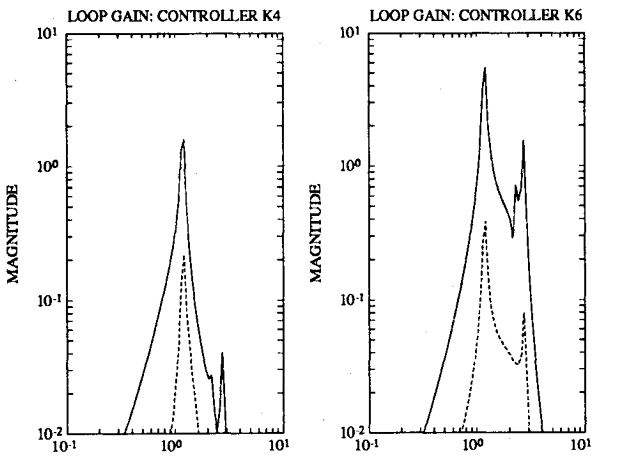

Fig. 9 Loop gain plots of controllers $K4$ and $K6$.

structure, it is seen that as the additive uncertainty weight is reduced, the torsional mode is attenuated by the controllers. This is due to the magnitude of the additive uncertainty weight not fully covering the first torsional mode.

Plots of the open-loop and closed-loop experimentally derived transfer functions for each controller are shown in Figs. 10–15. Attenuation of the first two bending and first torsional mode is substantial in control design $K3$ (Fig. 12). The experimental results show that controllers $K1$, $K2$, and $K3$, synthesized based on the six-mode model of the structure, rely on information about the first

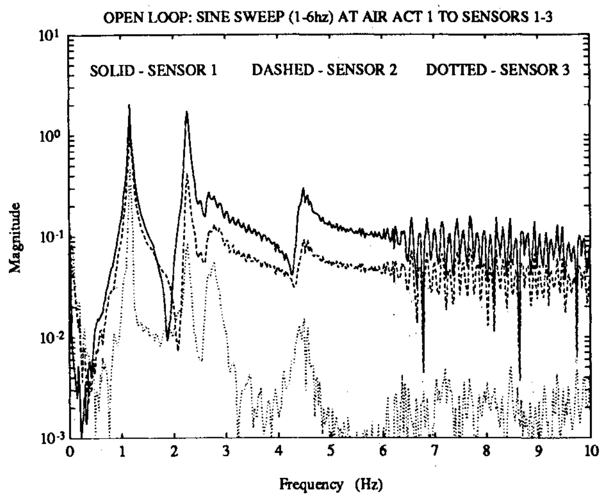

Fig. 10 Open-loop frequency response of experimental structure.

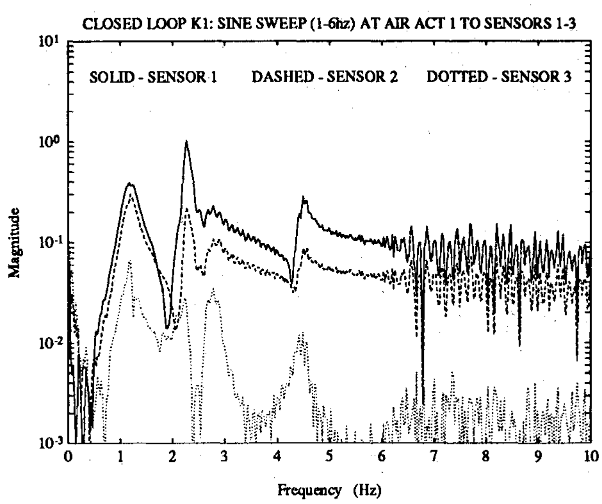

Fig. 11 Closed-loop frequency response of experimental structure, *K*1.

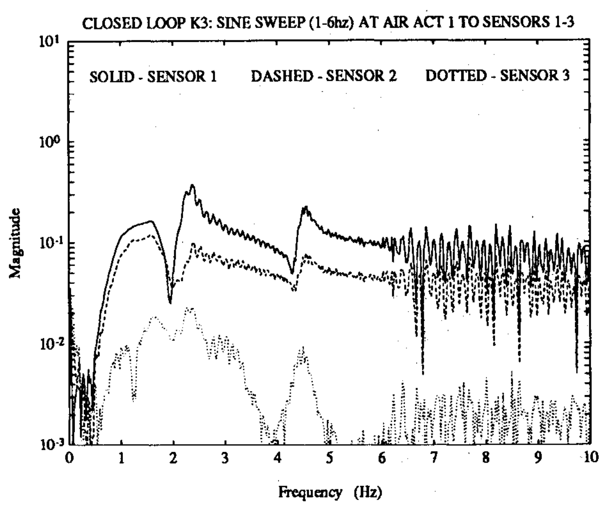

Fig. 12 Closed-loop frequency response of experimental structure, *K*3.

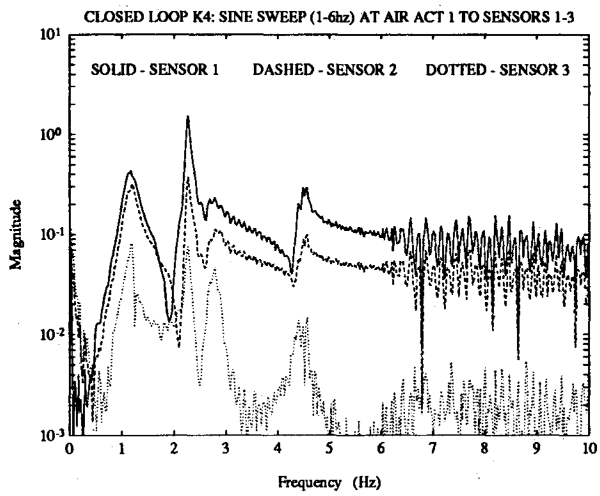

Fig. 13 Closed-loop frequency response of experimental structure, *K*4.

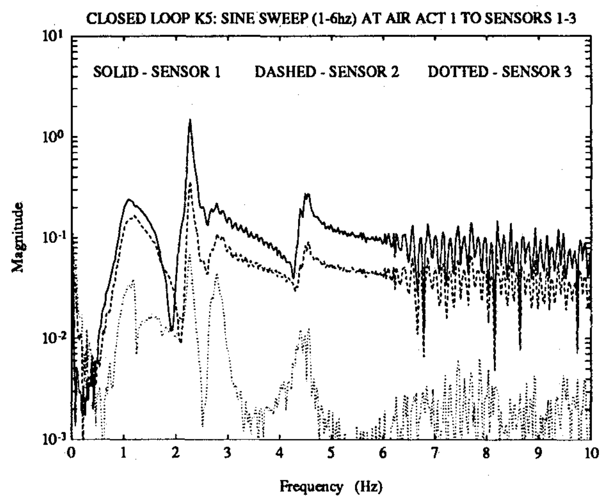

Fig. 14 Closed-loop frequency response of experimental structure, *K*5.

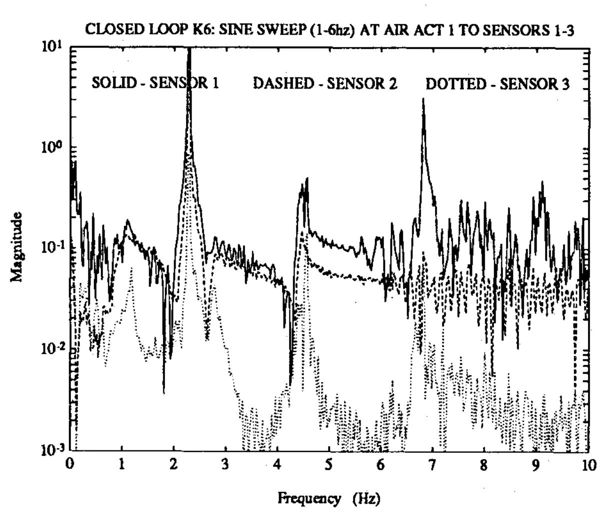

Fig. 15 Closed-loop frequency response of experimental structure, *K*6.

torsional and higher frequency structural modes to achieve the desired objectives. Controllers $K1$, $K2$, and $K3$ also attenuate the response of the first torsional mode.

One would speculate that if models of the higher frequency modes were inaccurate and the magnitude of the additive uncertainty weight was selected incorrectly, these controllers may destabilize higher frequency modes. A second set of controllers is synthesized with only a model of the first two bending modes of the experimental structure to investigate this conjecture. Loop gain plots for controllers $K4$ and $K6$ are shown in Fig. 9. Controller $K4$ is synthesized based on the nominal additive uncertainty weight. It achieves a closed-loop performance level on the experimental structure similar to that of controller $K1$ (see Table 2 and Figs. 11 and 13). This is expected since the additive uncertainty weight accurately accounts for the four neglected structural modes. Controller $K4$ attenuates the first two bending modes then rolls off fast enough to gain stabilize the first torsional mode.

The additive uncertainty weight is reduced by 50% to design controller $K5$. The closed-loop system (which includes the six-mode structural model) with controller $K5$ is analyzed using μ. Figure 8 shows that $K5$ does not destabilize or affect the response of the first torsional or high modes since the robust stability μ is less than 1. Experimental results are shown in Fig. 14. It is serendipitous that the first torsional mode is not destabilized since its magnitude is not fully covered by the scaled additive uncertainty weight to insure it is gain stabilized by the controller.

The additive uncertainty is reduced to 10% of its original value in the design of controller $K6$. The robust stability and robust performance plots of the closed-loop system with the six-mode structural model is shown in Fig. 8. Although the first torsional mode is not destabilized, corresponding to robust stability μ of less than 1, the closed-loop performance is severely degraded. This is seen in the robust performance μ plot for $K6$ and in the closed-loop experimental data in Figs. 8 and 15, respectively. This is in spite of the fact that the first two bending modes are heavily attenuated. Notice that in Fig. 8 $K6$ achieves the robust stability specification. The loop gain plot of $K6$ in Fig. 9 indicates that the higher frequency modes of the structure are not gain stabilized.

These results indicate that controllers can be synthesized using D-K iteration that achieves vibration attenuation objectives on lightly damped, flexible modes extremely closed to other lightly damped modes gain stabilized by the controller. Achieving these objectives requires the correct selection of uncertainty weights to account for the modes to be gain stabilized.

Conclusion

A number of controllers are designed that roll off in a region of numerous flexible modes and that achieve a specified level of attenuation in the controlled modes. An additive uncertainty weight is used in the problem formulation to require that the control laws gain stabilize the first torsional mode of the experimental structure. Controllers synthesized using the full six-mode model take advantage of knowledge of the higher structural modes to achieve improved vibration attenuation of these modes in the design. This is very evident as the size of the additive uncertainty weight is reduced.

Control laws synthesized using only the first two bending modes provided significant attenuation of these modes without affecting the first torsional mode. In fact, the first torsional mode has the same frequency domain peak for the open-loop and closed-loop cases when it is not destabilized. It is seen that an inadequate uncertainty representation of unmodeled high frequency modes, as seen in the design of controller $K6$, can lead to severe performance degradation or instability of these modes.

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